

# Solving Systems of Equations Using Inverse Matrices

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## Step 1:

Use matrix multiplication to multiply the matrices in the equation below.

$$\begin{bmatrix} 5 & -4 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix} \quad \begin{array}{l} 5x - 4y = 8 \\ x + 2y = 6 \end{array}$$

You get a system of Equations!

## Step 2:

Working backward from step 1, we can write a system of equations as a matrix equation.

$$\begin{array}{l} 2x - y = -10 \\ -3x + 4y = 5 \end{array} \quad \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -10 \\ 5 \end{bmatrix}$$

## Step 3:

Now we can solve the matrix equation using an inverse matrix!

$$A^{-1} \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \begin{bmatrix} -10 \\ 5 \end{bmatrix}$$

Multiply both sides of the equation by the inverse of the 2x2 matrix:

*Det*  $8 - 3 = 5$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} -10 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} -40 + 5 \\ -30 + 10 \end{bmatrix} = \begin{bmatrix} -35 \\ -20 \end{bmatrix}$$

so ...  $x = -7$  and  $y = -4$  !

Here are some for you to try ...

Example 1:

Solve the system of equations using an inverse matrix.

$$\begin{aligned} 2x + 5y &= 19 \\ 3x + 2y &= 1 \end{aligned}$$

- standard form

Det  $4 - 15 = -11$

$$\begin{bmatrix} 2 & 5 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19 \\ 1 \end{bmatrix}$$

- Matrix equation

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-11} \begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 19 \\ 1 \end{bmatrix}$$

- 2x2 inverse

$$\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{11} \begin{bmatrix} 33 \\ -55 \end{bmatrix}$$

- multiply matrices

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 5 \end{bmatrix} \quad \boxed{\begin{aligned} x &= -3 \\ y &= 5 \end{aligned}}$$

Example 2:

Solve the system of equations using an inverse matrix.

Clear the fractions + rearrange in standard form

$$\begin{aligned} 5x + 7y &= 9 \\ 3\left(y = -\frac{2}{3}x + 1\right) &\Rightarrow 3y = -2x + 3 \Rightarrow \boxed{2x + 3y = 3} \end{aligned}$$

Det  $15 - 14 = 1$

$$\begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1} \begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 9 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \end{bmatrix} \quad \boxed{\begin{aligned} x &= 6 \\ y &= -3 \end{aligned}}$$