Solving Systems of Equations Using Inverse Matrices

Step 1:

Use matrix multiplication to multiply the matrices in the equation below.

$$\begin{bmatrix} 5 & -4 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

You get a system of Equations!

Step 2:

Working backward from step 1, we can write a system of equations as a matrix equation.

$$2x - y = -10$$
$$-3x + 4y = 5$$

$$\begin{bmatrix} & \\ & \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix}$$

Step 3:

Now we can solve the matrix equation using an inverse matrix!

$$\begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -10 \\ 5 \end{bmatrix}$$

Multiply both sides of the equation by the inverse of the 2x2 matrix:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{\square} \begin{bmatrix} & \\ & \end{bmatrix} \cdot \begin{bmatrix} -10 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix}$$

so … x = _____ and y = _____ !

Here are some for you to try …

Example 1:

Solve the system of equations using an inverse matrix.

$2x + 5y = 19$
$3x + 2y = 1$

Example 2:

Solve the system of equations using an inverse matrix. Hint: Clear the fractions & rearrange in standard form

$5x + 7y = 9$
$y = -\frac{2}{3}x + 1$